LBNL-43486

The relationship between particle freeze-out distributions and HBT radius parameters

D. Hardtke[(a)] and S.A. Voloshin[(a,b)]
(a) Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720
(b) Department of Physics and Astronomy, Wayne State University, Detroit, MI 48201
(October 1, 1999)

Abstract

The relationship between pion and kaon space-time freeze-out distributions and the HBT radius parameters in high-energy nucleus-nucleus collisions is investigated. We show that the HBT radius parameters in general do not reflect the R.M.S. deviations of the single particle production points. Instead, the HBT radius parameters are most closely related to the curvature of the two-particle space-time relative position distribution at the origin. We support our arguments by studies with a dynamical model (RQMD 2.4).

I. INTRODUCTION

Two-particle intensity interferometry (HBT) has been used extensively in nucleus-nucleus collisions to measure the space-time extent of the particle emitting region [1]. Because of the complicated dynamics of high-energy nucleus-nucleus collisions, the HBT radii, defined as parameters of a Gaussian fit to the correlation function, do not measure the full size of the particle emitting region directly. Instead, the HBT radius parameters reflect an interplay between the geometric system size, the expansion of the system, the duration of particle emission, and the contribution of particles from resonance decays. It is hoped that the two-particle correlation functions can be parameterized in such a way that the various factors that influence the measured HBT radii can be understood individually.

The two-particle correlation function is typically measured as a function of momentum difference $q = p_1 - p_2$ and parameterized in term of radius parameters R_i and the so-called haoticity parameter λ:[1]

$$C_2(q) = 1 + \lambda e^{-q_i^2 R_i^2}. \tag{1.1}$$

For a static Gaussian pion source, the HBT radius parameters R_i directly correspond to the variance of the particle spatial distribution at freeze-out [1],

$$R_i^2 = RMS^2_{(x_i - V_i t)} = \sigma^2(x_i - V_i t), \tag{1.2}$$

where V_i is the particle velocity and R_i are the HBT radius parameters obtained from a fit to the correlation function. For a non-Gaussian source, this simple relationship no longer holds. In this paper, we explore the meaning of the Gaussian radius parameters in the case of a non-Gaussian particle source and use a dynamical model to investigate the breakdown of equation 1.2.

II. BASIC RELATIONS

Note that in general case of the particle source function the relation (1.2) *cannot* be true. A simple example would be the source with added a few particles emitted at very large distances. The addition of such particles can significantly change the freeze-out distribution R.M.S value and at the same time should not change the correlation function (sensitive only to pairs with close production points). Below we argue that the correlation function, and in particular the HBT radii derived from the Gaussian fit to the correlation function are indeed sensitive only to the distribution of close pairs. Hence, they can be quite different from the R.M.S. values of particle freeze-out distribution.

Let us show that the Gaussian fit to the correlation function yields the value of $\langle q_i^2 \rangle = (2R_i^2)^{-1}$ where the average is taken over the function $C_2(q) - 1$. Consider a Gaussian fit to a one-dimensional correlation function using the maximum likelihood method. The normalized fit function is

---

[1] For simplicity, we omit in this parameterization the possible cross-terms [1], which are not important for our study of particle production at *mid-rapidity* and in *central* collisions.

$$f(q) = \frac{2}{\sqrt{\pi}} R e^{-q^2 R^2}. \tag{2.1}$$

The likelihood functional to be minimized is then

$$\ln L = \ln \prod_i f(q_i) = \ln \prod_i (\frac{2}{\sqrt{\pi}} R e^{-q_i^2 R^2}) = \sum_i (\ln \frac{2}{\sqrt{\pi}} + \ln R - q_i^2 R^2) \tag{2.2}$$

The likelihood equation becomes,

$$\frac{\partial \ln L}{\partial (R^2)} = \sum_i (\frac{1}{2R^2} - q_i^2) = 0. \tag{2.3}$$

Solving this equation yields,

$$\frac{1}{2R^2} = \frac{\sum_i q_i^2}{N} = \langle q^2 \rangle. \tag{2.4}$$

Hence, irrespective of the shape of the measured correlation function, the fitted Gaussian HBT radii measure the value of $\langle q^2 \rangle$.

$\langle q^2 \rangle$ can also be related to the shape of the source function (time integrated relative distance distribution) $S(\Delta \mathbf{r})$, where $\Delta \mathbf{r}$ is the distance between particle production points $r_i = x_i - V_i t$. Using this function, the correlation function in momentum space can be Fourier transformed into position space [2,3]:

$$C(q) - 1 = \int e^{-i\mathbf{q}\Delta\mathbf{r}} S(\Delta \mathbf{r}) d\Delta\mathbf{r} \tag{2.5}$$

$$S(\Delta \mathbf{r}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{q}\Delta\mathbf{r}} (C(q) - 1) d\mathbf{q}. \tag{2.6}$$

Differentiating the source function twice yields,

$$\frac{1}{S(\Delta \mathbf{r})} \left. \frac{\partial^2 S(\Delta \mathbf{r})}{\partial \Delta r_i^2} \right|_{\Delta \mathbf{r} = 0} = -\langle q_i^2 \rangle, \tag{2.7}$$

where the average is taken over the correlation function, $C(q) - 1$. Eqns. 2.4 and 2.7 show that the HBT radius parameters are directly related to the second derivative of the source function of relative production points near $\Delta \mathbf{r} = 0$. This is true irregardless of the shape of the correlation function.

Note that the lambda parameter in the fit function (Eq. 1.1) in a maximum likelihood approach is defined by the normalization

$$\int d\mathbf{q} \lambda e^{-q_i^2 R_i^2} = \int (C(q) - 1) d\mathbf{q}. \tag{2.8}$$

The r.h.s. of this expression can be determined from equation 2.6:

$$\int (C(q) - 1) d\mathbf{q} = (2\pi)^3 S(0). \tag{2.9}$$

It follows then that

$$\lambda = 2^3 \pi^{3/2} S(0) R_1 R_2 R_3, \tag{2.10}$$

where $S(0)$ is the (true) source function at the origin, and R_1, R_2, and R_3 are the (fitted) radius parameters.

III. MODEL CALCULATIONS

To understand the relationship between the fitted HBT radii and the source function of relative production points and check how well the above equations work for a realistic particle source, the event generator RQMD (v2.4) [4] is employed. This model has been shown to reproduce experimental HBT data reasonably well [5]. We focus on particle production in midrapidity region ($|y| < 0.5$) in central ($b < 3$fm) Au+Au collisions at RHIC energy ($\sqrt{s} = 200$ GeV/nucleon).

Figure 1 shows single particle freeze-out position distribution and the two-particle relative position distribution for pions. The single particle freeze-out position difference distribution has long non-Gaussian tails. In addition, the shape is not well described by a Gaussian at $r = 0$. The two-particle position difference distribution still shows large non-Gaussian tails. The distribution, however, is well described as a Gaussian near $\Delta\mathbf{r} = \mathbf{0}$. Fitting this distribution to a Gaussian (also shown in the Figure) yields a Gaussian width σ that is substantially smaller than the R.M.S. deviation of the distribution.

The two-particle correlation functions are calculated as a function of p_T for mid-rapidity ($|y| < 0.5$) pions and kaons assuming plane wave propagation for the outgoing particles. The three-dimensional correlation function is then fit using the Bertsch-Pratt out-side-long parameterization,

$$C_2(q_o, q_s, q_l) = 1 + \lambda e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2}, \tag{3.1}$$

where q_l is the momentum difference along the beam axis, q_o is the momentum difference perpendicular to the beam axis and parallel to the total transverse momentum of the pair, and q_s is the momentum difference perpendicular to the beam axis and perpendicular to the total transverse momentum of the pair.

Figure 2 shows the fitted HBT radius parameters as a function of transverse momentum. Also plotted are the R.M.S. deviations of the single particle freeze-out distributions and the fitted Gaussian widths of the two-particle position difference distributions. The R.M.S. deviations are substantially larger than the fitted HBT radii in all cases, while the fitted Gaussian widths of the two-particle relative position difference distributions are very close to the fitted HBT radii. The single-particle R.M.S. deviation differs from the fitted HBT radii most at low transverse momentum. This deviation arises from the long non-Gaussian tails in the freeze-out position distribution due to pions from resonance decay and particular expansion dynamics of the system.

The width parameter (sigma) of the Gaussian fit to the two-particle difference distribution near the origin (in our calculations we perform the fit in the region of ± 0.5 R.M.S.) is a very good estimate of the second derivative of the relative source function at $\Delta\mathbf{r} = \mathbf{0}$. Hence, the observed good agreement between HBT radii (fit parameters to the correlation function) and widths of the Gaussians fitted to the two-particle position difference distribution near the origin confirm relationship shown in eqns. 2.4 and 2.7. Note a simple interpretation of the lambda parameter following from Fig. 1 (two-particle position difference distributions). It is just a product of the ratios of area under the Gaussian fit and the total number of pairs used for the distribution.

Figure 3 is similar to figure 2 except that the particles used are kaons. In this case, the R.M.S. deviations of the single-particle freeze-out distributions and the fitted Gaussian width of the two-particle relative distance distribution are very similar in the transverse directions. Since fewer kaons come from resonance decays, the long non-Gaussian tails in the single particle freeze-out distributions are greatly reduced. In the longitudinal direction, where the non-Gaussian nature of the production point distribution is mostly defined by the system expansion dynamics (very similar for pions and kaons), the difference between the HBT radii and the R.M.S. values of freeze-out distribution is quite significant. The lambda parameter in the kaon case is generally larger than that for the pion case but due to a limited statistics exhibits large fluctuations and is omitted from the figure.

IV. CONCLUSIONS

We have shown that the HBT radius parameters obtained from a Gaussian fit to the correlation function are equivalent to the the curvature of the relative freeze-out distributions at $\Delta\mathbf{r} = \mathbf{0}$. Therefore, the fitted HBT radii are directly related to fitted Gaussian width of the two-particle position difference distribution near the origin. The lambda parameter is determined by the ratio of the value of the real relative distance distribution at the origin and the distribution chosen in a Gaussian form with the radii parameters determined by the fit to the shape of the correlation function. Studies with a dynamical model support the above statements.

ACKNOWLEDGMENTS

We would like to thank Heinz Sorge for the use of his RQMD code, and Scott Pratt for the use of the CRAB correlation after-burner. In addition we would like to thank Nu Xu for providing the RQMD events and useful discussions and U. Heinz for comments.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics of the U.S. Department of Energy under Contracts DE-AC03-76SF00098 and DE-FG02-92ER40713.

REFERENCES

[1] For a recent reviews, see U. Heinz and B.V. Jacak, Ann. Rev. Nucl. Part. Sci. 49 (1999); U..A. Wiedemann and U. Heinz, preprint nucl-th/9901094, submitted to Phys.Rep.

[2] D.A. Brown and P. Danielewicz, Phys. Rev. C **57**, 2474 (1998); Phys. Lett. **B398**, 252 (1997).

[3] U.A. Wiedemann and U. Heinz, Phys. Rev. C **56**, R610 (1997).

[4] H. Sorge, H. Stöcker, and W. Greiner, Nucl. Phys. **A498**, 567c (1989); Ann. Phys. **192**, 266 (1989).

[5] J. P. Sullivan, M. Berenguer, B.V. Jacak, M. Sarabura, J. Simon-Gillo, H. Sorge, H. van Hecke, and S. Pratt, Phys. Rev. Lett. **70**, 3000 (1993); D.E. Fields, J.P. Sullivan, J. Simon-Gillo, H. van Hecke, B.V. Jacak, and N. Xu, Phys. Rev. C **52**, 986 (1995). I.G. Bearden, *et al.* (NA44 Collaboration), Phys. Rev. C bf 58, 1656 (1998).

FIGURES

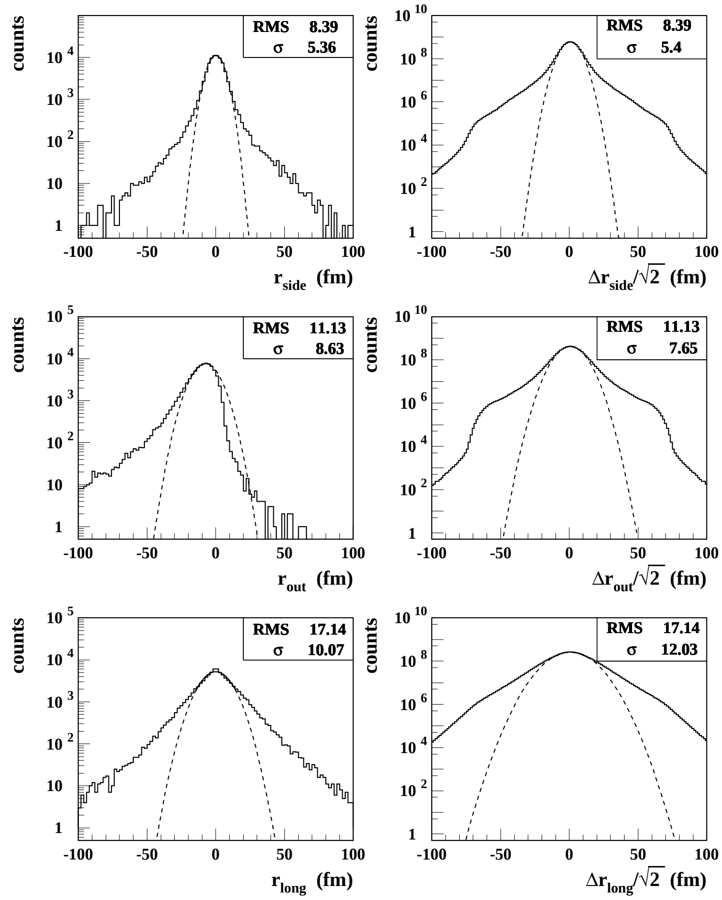

FIG. 1. The single particle freeze-out position distributions and two-particle position difference distributions for pions with $0 < p_T < 200$MeV/c and $|y| < 0.5$ for r_{side} (perpendicular to the transverse momentum vector), r_{out} (parallel to the transverse momentum vector), and r_{long} (along the beam axis). The Gaussian fit function is also shown in each case, along with the fit parameters.

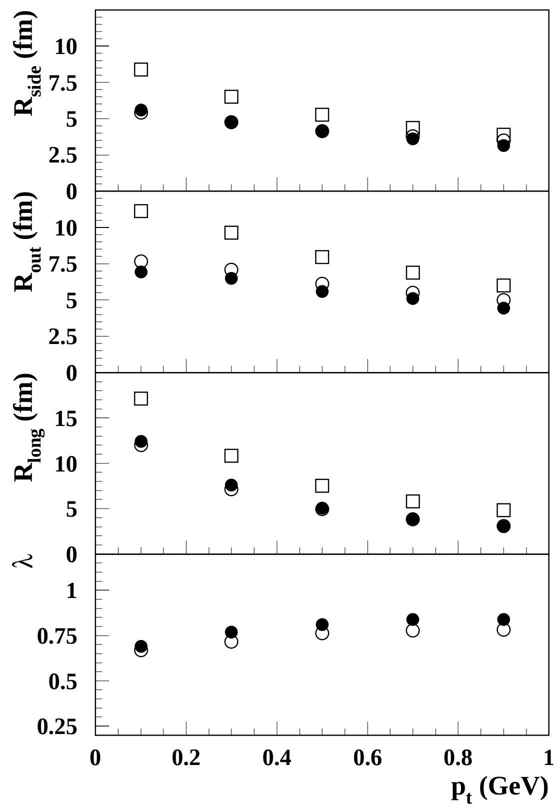

FIG. 2. The fitted HBT radius parameters (solid circles), single-particle R.M.S. deviations (open squares), and the sigma of the Gaussian fit two-particle position difference distribution (open circles) as a function of p_t for R_{side}, R_{out}, R_{long}. The rapidity range is $|y| < 0.5$. The bottom plot show the lambda parameter determined from the fit to the correlation function (solid circles) and from the two particle position difference distribution.

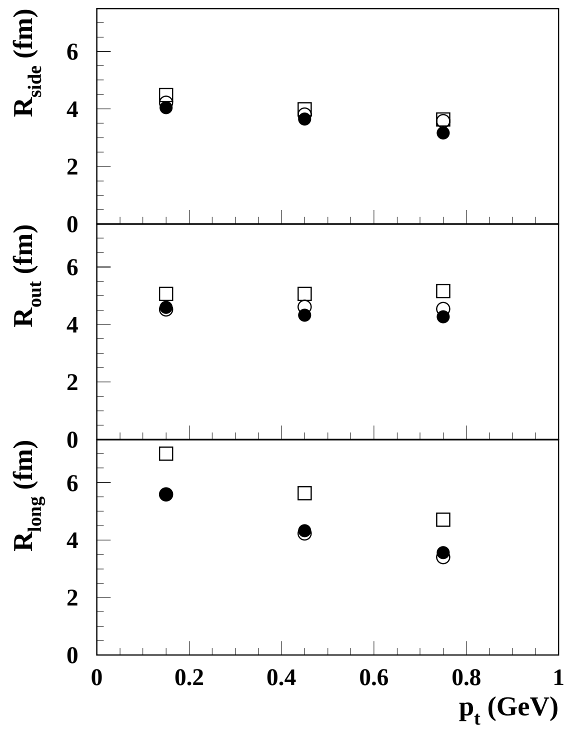

FIG. 3. The same as Fig. 2, but for kaons.